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Research Article

Modelling the Control of the Impact of Fall Armyworm (*Spodoptera frugiperda*) Infestations on Maize Production

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In this paper, we propose and analyze a stage-structured mathematical model for modelling the control of the impact of Fall Armyworm infestations on maize production. Preliminary analysis of the model in the vegetative and reproductive stages revealed that the two systems had a unique and positively bounded solution for all time $t \geq 0$. Numerical analysis of the model in both stages under two different cases was also considered: Case 1: different number of the adult moths in the field assumed at $t = 0$ and Case 2: the existence of exogenous factors that lead to the immigration of adult moths in the field at time $t > 0$. The results indicate that the destruction of maize biomass which is accompanied by a decrease in maize plants to an average of 160 and 142 in the vegetative and reproductive stages, respectively, was observed to be higher in Case 2 than in Case 1 due to subsequent increase in egg production and density of the caterpillars in first few (10) days after immigration. This severe effect on maize plants caused by the unprecedented number of the pests influenced the extension of the model in both stages to include controls such as pesticides and harvesting. The results further show that the pest was significantly suppressed, resulting in an increase in maize plants to an average of 467 and 443 in vegetative and reproductive stages, respectively.

1. Introduction

Food loss due to Fall Armyworm (FAW-*Spodoptera frugiperda*) is currently one of the biggest threats to food security, particularly in large parts of the developing world. As reported by the United Nations [1] and Shiferaw et al. [2], the world's population is expected to reach 9.3 billion by the end of 2050, with an approximate yearly increase of more than 80% of the global increase, and a quarter of this increase is expected to occur in developing countries. This unprecedented global increase in the number of people poses a serious challenge for maize producers and policymakers, especially regarding the minimization of food losses due to the effect of interaction between maize crops and FAW. This issue has been a matter of active research for many years, with the main challenge lying in the unavoidable trade-off between the reduction of FAW, the financial costs involved, and the environmental impacts.

The severity and extent of FAW outbreaks are enhanced following the onset of the wet season when the wind-borne immigrations of adult moths are attracted to lay eggs, which transform into caterpillars within 2 to 5 days [3]. The newly hatched caterpillars benefit from the flush of green maize vegetation resulting from the rain and develop rapidly over three weeks and outbreaks can have a very high density. The caterpillars can severely devastate maize plantations over several thousand square kilometres with a very high population density [4, 5]. These severe effects of FAW outbreaks particularly occur when rainstorms follow droughts [3]. The damage of whorl maize leaves by the caterpillars as they grow leaves semitransparent patches called windows, which interfere with the growth of maize plants.

According to De Groote, Faithpraise, Anguelov, Kebede, Paez, and Pearce [4–9], the FAW outbreaks started in Nigeria in 2016 from where they spread to the nearby Sub-Saharan African countries, creating up to 90% maize crop losses. These outbreaks travelled over thousands of kilometres to East African countries such as Tanzania (January 2017), Kenya (April 2017), and Uganda (May 2017) [4]. Numerous studies have been conducted to understand these periodic outbreaks. Mathematical models based on weather patterns and estimated dates of arrival of the moths from rainfall forecasting and caterpillar counts from field inspections have also been formulated to predict possible outbreaks [10–12].

In a recent study, the mitigation of the damaging effects of FAW was performed using control measures such as treating the larvae with insecticides such as azadirachtin, synthetic pesticides such as DDT, and aqueous neem seed extracts [6]. The success was dose-dependent and produced a range of undesirable adverse side effects on sensitive wildlife and human health [13]. Paez [8] used biological control for the African Armyworm, which also can be applied to FAW, and Liang [13] used nuclear polyhedrosis virus (NPV), which kills armyworms and helps control outbreaks. Murua [14] proposed the control of the FAW using larvae parasites. The system was very reliable, since 95% of the pest larvae were destroyed but the system was unable to reduce the pest eggs to a sufficiently low level to maintain control.

There exists a rich set of mathematical models of FAW effects on maize production which were reported by several authors [6, 8, 15]. Models like stage-structured with and without time delay which often considered mature predators (natural enemy) have also been applied; however, most of these models, despite incorporating Holling types I, II, III, and IV and Beddington-DeAngelis functional responses as their baseline, did not target the interaction between maize and FAW [8, 16–24].

Considering the importance of maize to a majority of countries producing maize in Africa, the present work aims to utilize ordinary differential equation in exploring the implications of FAW infestation in a maize field planted with initial number of maize seeds at time $t = 0$ and obtaining maximum harvest at the end of the season. To come up with the intended results, we propose two subgeneric models, each with stage-structured in both of populations (maize and FAW) to determine the population dynamics in the presence and absence of immigration of the adult moth and estimate the yield when control measures such as pesticides and harvesting are deployed.

2. Model Formulation

The two submodels introduced herein consist of two populations: maize and FAW, both of which are stage-structured giving a total of five populations. We consider maize growth from emergence to maturity at any given time $t > 0$, growing through two time periods: Period I and Period II. Period I, which is assumed to take place over a time period $[0, t_1]$, denotes the vegetative stage and comprises planting of maize seeds, seed emergence, development of whorl leaves, and tasselling, while period II, which takes place over a time period $[t_1, t_2]$, denotes the reproductive stage and consists of corncob, kernel development, and maturity.

On the other hand, the FAW population at any time $t > 0$ has been subdivided into egg population, caterpillar population, and the adult moth population. Although the FAW has six larval instar stages, we have considered this as single group called caterpillar in order to reduce complexity of the model. We assume that weather condition, environment condition, and planting system of maize seeds favor seed germination and their corresponding growth in both stages with no natural death rate before harvest.

In this regard, we let $x_1(t)$ represent the population density of maize in the vegetative stage and let $x_2(t)$ represent the population density of maize in the reproductive stage, while $w(t)$, $y(t)$, and $z(t)$ represent the population density of eggs, caterpillars, and the adult moths, respectively. We also assume that caterpillar with a mortality rate μ_y is the only threat to maize throughout its growth period and the adult moth takes over in the reproduction process. When food is limited, the older caterpillar of FAW exhibits a cannibalistic behavior on the smaller larvae [25, 26].

The population density of eggs is replenished through the laying of eggs by the adult moth at a constant rate ρ per day and reduced through hatching into caterpillar and destruction (mortality) at rates γ and μ_w, respectively. On the other hand, the adult moth's population density is refilled through the caterpillar's development into an adult moth at a constant rate δ and reduced through mortality at a rate μ_z. We further assume that, in a season (i.e., Period I and Period II), there is no maize seed replantation and the model in both stages has no maize recruitment. Therefore, the classes $x_1(t)$ and $x_2(t)$ decline constantly due to caterpillar attack at the rates of α and η, respectively, with a destruction rate λ in each stage. The study assumes nonnegative values of the model parameters and variables in context with populations being considered. The formulation of this model is also supported by the following assumptions:

(i) Planting of maize seed is done at $t = 0$; therefore, the development rate of each maize plant from the vegetative stage to the reproductive stage is the same and continuous.

(ii) At $t = 0$, $x_1(0) = k$, where k represents the maximum number of maize plants the field under consideration can have.

(iii) Assume that the only source of food for the caterpillar is maize, so that, in its absence, caterpillar becomes extinct.

(iv) The number of maize plants in a garden cannot exceed k as $t \longrightarrow T$, where T represents time to maturity stage.

The summary of the definitions of model state variables and parameters is given in Tables 1 and 2, respectively.

Table 1: Description of the model state variables used in the model.

Variables	Description
$x_1(t)$	Population density of the maize plants growing in the vegetative stage at any time t
$x_2(t)$	Population density of the maize plants growing in the reproductive stage at any time t
$y(t)$	Population density of the caterpillars at any time t
$z(t)$	Population density of adult moths at any time t
$w(t)$	Population density of the eggs laid at any time t

Table 2: Descriptions of the parameters used in the model.

Parameter	Description
α	The rate at which caterpillars attack $x_1(t)$
η	The rate at which caterpillars attack $x_2(t)$
ρ	Eggs-laying rate
k	Maximum number of maize plants the garden under consideration can have at $t = 0$
δ	The rate at which the caterpillars develops into an adult moth
γ	The rate at which the eggs hatch into caterpillar species
μ_y	Caterpillar's death rate
μ_z	Adult moth's death rate
μ_w	Eggs' death rate
λ	The rate at which maize dies due to caterpillar attack

The model explanations above can be represented schematically as shown in Figure 1.

The model described in Figure 1 is transformed into two interconnected or coupled model systems for the vegetative and reproductive stages. The two systems at the transitional period (i.e., $t = t_1$) are connected by the initial condition $x_1(t_1) = x_2(t_1)$, where $x_1(t_1)$ represents the number of maize plants in the vegetative stage progressing into the reproductive stage, while $x_2(t_1)$ represents the number of maize plants that have progressed into the reproductive stage.

The model in vegetative stage $(0 \le t \le t_1)$ is

$$\begin{aligned}\frac{dx_1}{dt} &= -\alpha x_1 y - \lambda x_1,\\ \frac{dy}{dt} &= e_1 \alpha x_1 y + \gamma w - \delta y - \mu_y y,\\ \frac{dz}{dt} &= \delta y - \mu_z z,\\ \frac{dw}{dt} &= \rho z - \gamma w - \mu_w w,\end{aligned} \tag{1}$$

with initial conditions:

$$\begin{aligned} x_1(0) &= k,\\ x_2(0) &\ge 0,\\ y(0) &\ge 0,\\ z(0) &\ge 0,\\ w(0) &\ge 0.\end{aligned} \tag{2}$$

The model in reproductive stage $(t_1 \le t \le t_2)$ is

$$\begin{aligned}\frac{dx_2}{dt} &= -\eta x_2 y - \lambda x_2,\\ \frac{dy}{dt} &= e_2 \eta x_2 y + \gamma w - \delta y - \mu_y y,\\ \frac{dz}{dt} &= \delta y - \mu_z z,\\ \frac{dw}{dt} &= \rho z - \gamma w - \mu_w w,\end{aligned} \tag{3}$$

with initial conditions:

$$\begin{aligned} x_2(t) &= 0, \quad \text{for } t < t_1,\\ x_2(t_1) &= x_1(t_1).\end{aligned} \tag{4}$$

The parameters e_1 and e_2 are the conversion rates of maize biomass into caterpillar biomass in the vegetative and reproductive stages, respectively. The formulated systems (1) and (3) are interconnected such that the solutions at time $t = t_1$ of system (1) are the initial conditions for system (3). To prove this behavior, we first investigate the basic properties of the model systems as follows.

3. Basic Properties of the Models

The effect of interaction between maize and FAW is studied by analyzing systems (1) and (3) to determine if these models are mathematically and epidemiologically well posed. The study is carried out for maize growing in the vegetative and reproductive stages, which are model systems (1) and (3), respectively. We consider the basic properties of the model: positivity of solution and invariant region.

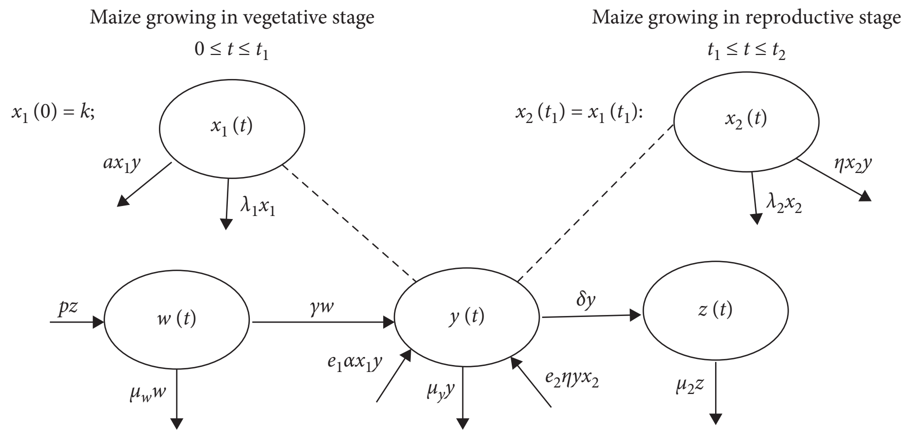


FIGURE 1: Model flow diagram illustrating the dynamics of FAW in a field of maize plants. The FAW life cycle is divided into three classes: egg stage $w(t)$, caterpillar $y(t)$, and adult moth stage $z(t)$. The compartment $x_1(t)$ represents maize plant in vegetative stage and $x_2(t)$ represents maize plant in reproductive stage. The dotted line demonstrates that FAW caterpillar is the one responsible for attacking the maize plant in both of the stages.

3.1. Positivity of Solution. For model systems (1) and (3) to be epidemiologically meaningful and well posed, it is required to prove that all solutions of these systems with their respective initial conditions remain positive for all $t > 0$. This is established by the following theorem.

Theorem 1. *Let* $\Omega_1 = \{(x_1, y, z, w) \in \mathfrak{R}^4_+ : x_1(0) = k, y(0) \geq 0, z(0) \geq 0, w(0) \geq 0\}$ *for system (1) and* $\Omega_2 = \{(x_2, y, z, w) \in \mathfrak{R}^4_+ : x_2(0) \geq 0, y(0) \geq 0, z(0) \geq 0, w(0) \geq 0\}$ *for system (3); then, the solution sets* $(x_1(t_1), y(t_1), z(t_1), w(t_1))$ *and* $(x_2(t_2), y(t_2), z(t_2), w(t_2))$ *for systems (1) and (3), respectively, are positive for all* $t > 0$ *in the interval* $[0, t_2]$.

Proof. In a biological sensible way $x_2(t) = 0$ in vegetative stage and $x_1(t) = 0$ in reproductive stage, to prove the theorem, we consider two cases: models in the vegetative stage and the reproductive stage.

Case 1. We consider system (1), the model in the vegetative stage $(0 \leq t \leq t_1)$.

Considering the first equation, $dx_1/dt = -\alpha x_1 y - \lambda x_1$. Grouping and arranging like terms, one gets

$$\frac{dx_1}{x_1} = -(\alpha y + \lambda)dt. \quad (5)$$

Integrating over a time interval 0 to t leads to

$$x_1(t) = x_0 e^{-\lambda t} e^{-\int_0^t (\alpha y(s))ds}, \quad (6)$$

where $x_0 = x_1(0)$ and, for real values of $y(t) \geq 0$ and $e^{-\int_0^t (\alpha y(s))ds} \geq 0$,

$$x_1(t) \geq 0, \ \forall\, 0 \leq t \leq t_1. \quad (7)$$

Similarly, from the second equation, $dy/dt = e_1 \alpha x_1 y + \gamma w - \delta y - \mu_y y$.

Setting

$$\theta(t) = e_1 \alpha x_1 - \delta - \mu_y, \quad (8)$$

with $x_1(t) \geq 0$, we get

$$\frac{dy}{dt} - \theta(t) y = \gamma w. \quad (9)$$

Applying integration with $w(t) \geq 0$, $\forall 0 \leq t \leq t_1$, we obtain $y(t) \geq 0$.

Continuing in the same manner, we can also show that $z(t) \geq 0$ and $w(t) \geq 0$, meaning that $x_1(t) \geq 0$, $y(t) \geq 0$, $z(t) \geq 0$, and $w(t) \geq 0$ exist in Ω_1 and the population densities for species denoted by the state variables in model (1) are nonnegative.

Case 2. For system (3), we consider the model in reproductive stage $0 \leq t \leq t_1$.

We can prove the first, second, third, and fourth equations of system (3) using the same techniques as in Case 1 and conclude that $x_2(t) \geq 0$, $y(t) \geq 0$, $z(t) \geq 0$, and $w(t) \geq 0$ exist in Ω_2, meaning that the population densities for species denoted by the state variables in system (3) are nonnegative. □

3.2. Invariant Region. In this section, we determine a region in which the solution of systems (1) and (3) is bounded. We use the box invariant of Metzler matrix method as applied by Aloyce [27] to show the existence of invariant regions. We prove this as follows: systems (1) and (3) can be written as

$$\frac{dX}{dt} = A(X)X + F, \quad (10)$$

with $X = (x_1, y, z, w)^T$ and the constant $F = (0, 0, 0, 0)^T$ for the model system (1) and

$$\frac{dY}{dt} = B(Y)Y + G, \quad (11)$$

with $Y = (x_2, y, z, w)^T$ and the constant $G = (0, 0, 0, 0)^T$ for the model system (3). From equation (10),

$$A(X)=\begin{bmatrix} -(\alpha y+\lambda) & 0 & 0 & 0 \\ e_1\alpha y & -\left(\delta+\mu_y\right) & 0 & \gamma \\ 0 & 0 & -\mu_y & 0 \\ 0 & 0 & \rho & -(\gamma+\mu_w) \end{bmatrix}, \qquad (12)$$

and, from equation (11),

$$B(Y)=\begin{bmatrix} -(\eta y+\lambda) & 0 & 0 & 0 \\ e_2\alpha y & -\left(\delta+\mu_y\right) & 0 & \gamma \\ 0 & 0 & -\mu_y & 0 \\ 0 & 0 & \rho & -(\gamma+\mu_w) \end{bmatrix}. \qquad (13)$$

Referring to equations (12) and (13), it follows that $A(X)$ and $B(Y)$ are Metzler matrices for systems (1) and (3), respectively, $\forall X, Y \in \mathfrak{R}_+^4$ in which all off-diagonal terms are nonnegative with $F \geq 0$ and $G \geq 0$. Therefore, according to Aloyce [27], systems (10) and (11) are positive invariants in $\mathfrak{R}_+^4$. These results lead us to the conclusion that systems (1) for $0 \leq t \leq t_1$ and (3) for $t_1 \leq t \leq t_2$ with their respective initial conditions have bounded nonnegative solutions. Since the solution for systems (1) and (3) is well posed and bounded, the following section gives the numerical analysis to assess the impact of the interaction between maize plants, eggs, caterpillars, and the adult moths population throughout the time interval $[0, t_2]$.

4. Numerical Simulation

Numerical simulations for systems (1) and (3) are carried out to illustrate the impact of FAW infestation on maize using a set of reasonable parameter values, where some of the data were obtained from literature and others were estimated based on the idea given by Li [28] and Tumwiine [29]. The parameters are chosen following realistic ecological observations. Using MATLAB software in simulating system (1) for time $t \in (0, t_1)$, three cases are investigated with their respective maize yields. Case 1 concerns the effect of the interaction on maize when there are different numbers of the adult moths assumed at $t = 0$. Case 2 concerns the effects of the interaction when there is immigration of the adult moth at time $t \neq 0$. Case 3 concerns the effect of the interaction when deploying controls. For both cases with $x_1(0) = 500$, $y(0) = 0$, and $w(0) = 0$, the final solutions, that is, $x_1(t_1)$, $y(t_1)$, $z(t_1)$, and $w(t_1)$, for each number of the adult moths assumed will be used as an initial value in simulating system (3) for time $t \in (t_1, t_2)$. Therefore, before simulating these models, we first discuss the population dynamics of maize and FAW and then estimate and adopt some of the parameter values based on current literature.

4.1. Population Dynamics of Maize. Maize plants grow through two main stages in a season: the vegetative stage (emergence to the tasselling stage) and the reproductive stage (tasselling to maturity). Maize planted at the time in an environment that may contain planted seeds germinates in 0–7 days. According to Kebede [7], maize in the vegetative and reproductive stages takes 63–97 days, respectively, to reach maturity from emergence. However, other maize seeds take about 90 or 151 days to reach maturity [27]. The infestation of FAW starts when an adult moth migrates into maize plants and lays eggs which then hatch into caterpillars that damage maize leaves and kernel development in the vegetative and reproductive stages, respectively. This infestation occurs continuously, where the pest is endemic throughout the year. Systems (1) and (3) represent the interaction between maize plants and FAW. After the simulation of the model, it will be possible to identify the output of the maize plants at $t_1 = 63$ days and $t_2 = 160$ days.

4.2. Population Dynamics of FAW. The FAW is a holometabolous insect (undergoes complete metamorphosis including eggs, caterpillar, and adult moth) [7]. Only the adult moths that survive for 10 days (7 to 21 days) reproduce and migrate to another location [30]. When the adult moth immigrates into a maize field, it deposits most of her 100–200 eggs per mass. After a preoviposition period of 3 and 4 days of life, oviposition occurs for up to 3 weeks [7, 31]. The total egg production per adult moth averages 1500 to a maximum of 2000 in its lifetime.

The egg stage takes 2-3 days before hatching into a caterpillar in a season depending on the climatic conditions. The caterpillar that develops into the adult moth survives for about 14 and 30 days during the warm summer and cooler months, respectively. Maize plants in the vegetative and reproductive stages are susceptible to caterpillar attack, reducing their ability to manufacture food and in turn reducing yield. In Section 4.3, we estimate and adopt the parameters to be used in the simulation.

4.3. Parameters Estimation and Adoption. Due to the unavailability of real data for all parameters related to systems (1) and (3), the suppositional values of the different parameters have been considered as follows: according to Tumwiine and Daudi [29, 31], mortality/death rate is the reciprocal of the average of the life span of an organism, whereby the life span is the duration an organism survives before dying; that is,

$$\text{mortality rate} = \frac{1}{\text{life span}}. \qquad (14)$$

Since the duration of eggs, caterpillar, and adult moth is 2.5, 14, and 10 days, respectively, their natural mutation/transformation rates are 0.400, 0.0071, and 0.161 per day, respectively. The development period for an egg into caterpillar and caterpillar into adult moth averages 17.5 and 14 days, respectively. This implies that the development rates of eggs into caterpillars and caterpillars into adult moths are 0.58 and 0.071 per day, respectively. Finally, the adult moth lays up to 2000 eggs in 10 days of its lifetime, and because a cycle may take up to 30 or 60 days depending on the weather conditions, the adult lays 0.017 eggs per day. According to Li [28], the survival/development rate is calculated as

$$\text{development rate} = \frac{N_1}{N_2}, \tag{15}$$

with N_1 and N_2 representing the number of pests that progress into the next development stage and number of pests in the previous development stage, respectively. Table 3 summarizes the parameter values used in systems (1) and (3).

Considering the above data, we plot and analyze system (1) for assessing the impact of the interaction between maize and FAW from time $t = 0$ to time $t_1 = 63$ days and system (3) from time $t = 0$ to time $t_2 = 160$ days under the following cases.

4.4. Impact of Population Dynamics in the Absence of Immigration. The simulation of the model systems (1) and (3) under this section is done by assuming that, for each number of the adult moths considered in the field such as $z_1(0) = 15$, $z_2(0) = 30$, $z_3(0) = 45$, and $z_4(0) = 60$; $x_1(0) = 500$ is the initial number of maize seeds planted, while $y(0) = 0$ and $w(0) = 0$ define the initial numbers of caterpillars and eggs, respectively. In this regard, the simulation of these models is shown in Figure 2.

The simulation indicates that, for each number of adult moths assumed at $t = 0$, we observe an exponential decrease of the adult moth's population size before attaining its equilibrium value due to eggs-laying process and mortality rate (Figure 2(a)). Figures 2(b) and 2(c) demonstrate that the egg-laying process increases egg production to its maximum environment carrying capacity, a situation which also increases the density of caterpillars in the same manner due to hatching. The increase in caterpillar's population size for each number of adult moths assumed exponentially decreases the biomass of maize plants, an effect that reduces the efficiency of photosynthesis and in turn reduces the number of maize plants. The results for each number of the adult moths assumed at $t = 0$, egg production, caterpillar density, and their corresponding effects on maize plants are summarized in Table 4, where both data are taken at $t_1 = 63$ days.

Now, we use the results for the model system (1) displayed in Table 4 taken at $t_1 = 63$ as an initial value in simulating model system (3) from $t_1 = 63$ to $t_2 = 160$ days, and the simulation results are shown in Figure 3.

We also observe in Figure 3 that the adult moths, eggs, and caterpillars are at equilibrium but maize continues decreasing to a certain endemic level at $t_2 = 160$ days. The number of adult moths, egg population, and their corresponding effects on maize as seen from Figures 3(a)–3(d) are in Table 5.

4.5. Impact of Population Dynamics in the Presence of Immigration. In Section 4.4, the model systems (1) and (3) were simulated with an assumption about the number of maize seeds planted and a different number of adult moths prevailing at $t = 0$ with zero initial number of eggs and caterpillars population. This section assumes that there exist some exogenous factors (such as temperature, wind, and further human actions) that lead to the immigration of the adult moths in the maize field, whereby, at $t > 0$, $x_1(0) = 500$, $y(0) = 0$, $z(0) = 0$, and $w(0) = 0$, the initial population densities of the adult immigrant moths lay eggs after entering the field. To incorporate the migratory behavior of the pest in models (1) and (3) at time $t > 0$, we assume that there is no emigration of adult moths in both vegetative and reproductive stages after immigration. Therefore, we let σ be the immigration rate of the pest. Assuming no mortality rate during the immigration process, the number of adult moths in the field will abruptly change in a short period. The models of (1) and (3), after incorporation of immigration, appear as shown in model systems (16) and (18).

For $t \le t_1$,

$$\begin{aligned}
\frac{dx_2}{dt} &= -\alpha x_1 y - \lambda x_1, \\
\frac{dy}{dt} &= e_1 \alpha x_1 y + \gamma w - \delta y - \mu_y y, \\
\frac{dz}{dt} &= \delta y - \mu_z z + \sigma, \\
\frac{dw}{dt} &= \rho z - \gamma w - \mu_w w,
\end{aligned} \tag{16}$$

subject to

$$\begin{aligned}
x_1(0) &= k, \\
x_2(0) &\ge 0, \\
y(0) &\ge 0, \\
z(0) &\ge 0, \\
w(0) &\ge 0.
\end{aligned} \tag{17}$$

For $t_1 \le t \le t_2$,

$$\begin{aligned}
\frac{dx_2}{dt} &= -\eta x_2 y - \lambda x_2, \\
\frac{dy}{dt} &= e_2 \eta x_2 y + \gamma w - \delta y - \mu_y y, \\
\frac{dz}{dt} &= \delta y - \mu_z z + \sigma, \\
\frac{dw}{dt} &= \rho z - \gamma w - \mu_w w,
\end{aligned} \tag{18}$$

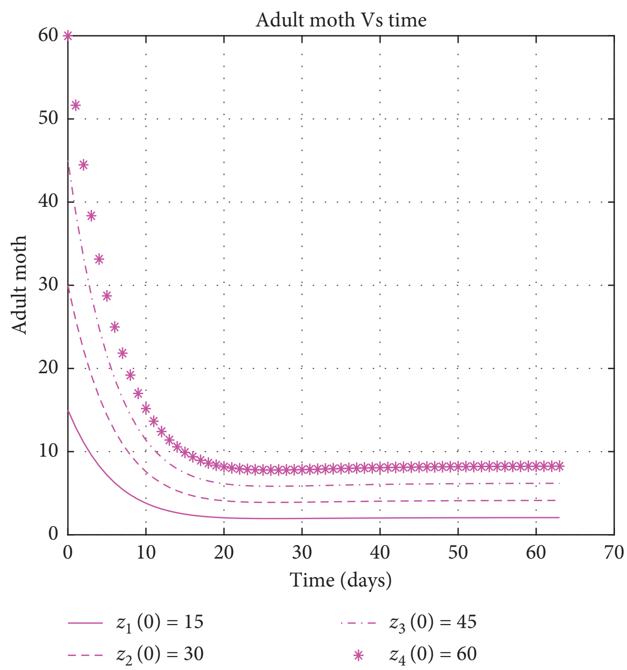
subject to

$$\begin{aligned}
x_2(t) &= 0, \quad \text{for } t < t_1, \\
x_2(t_1) &= x_1(t_1).
\end{aligned} \tag{19}$$

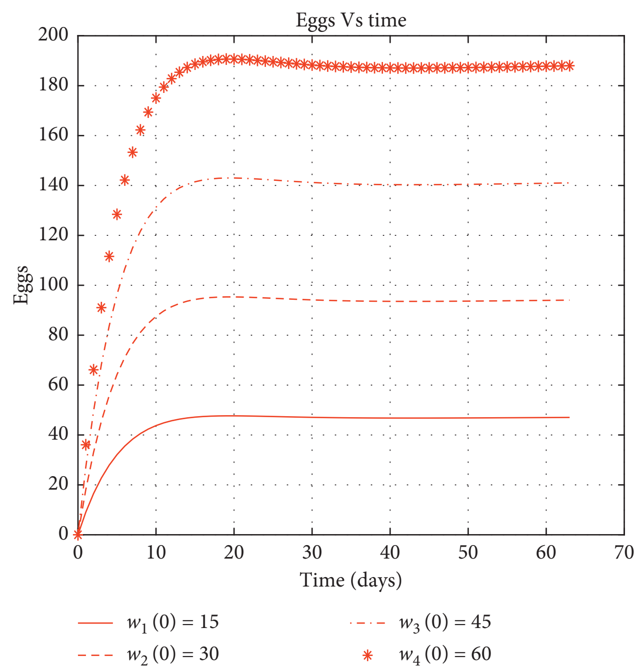
To examine the effects of immigration of the moth on the population dynamics of maize plants, models (16) and (18) with different values of immigration rates in the absence of the control measure are simulated. The adult moth on average lays eggs in batches of 100–200 per mass which hatch into caterpillars within 2–5 days. The caterpillar is a

Table 3: Parameter values for systems (1) and (3).

Parameter	Description	Values/unit	Reference
k	The maximum number of maize plants the garden under consideration can have at $t = 0$	500 plants	Assumed
α	The rate at which a caterpillar attacks $x_1(t)$	0.000154 plants/day	[32]
η	The rate at which a caterpillar attacks $x_2(t)$	0.000154 plants/day	[32]
ρ	Eggs-laying rate	0.0417 eggs/day	[32]
δ	The rate at which the caterpillars develop into adult moths	0.071 per/day	[4]
γ	The rate at which eggs hatch into caterpillars	0.071 per/day	[4]
μ_y	The death rate of caterpillars	0.0071 per/day	[4]
μ_z	The death rate of adult moths	0.115 per/day	[29]
μ_w	The death rate of eggs	0.04 per/day	[29]
λ	The rate at which maize plants die due to FAW attack	0.015 per/day	[30]
e	The conversion factor of maize biomass into caterpillar biomass	1.6 leaves	[30]

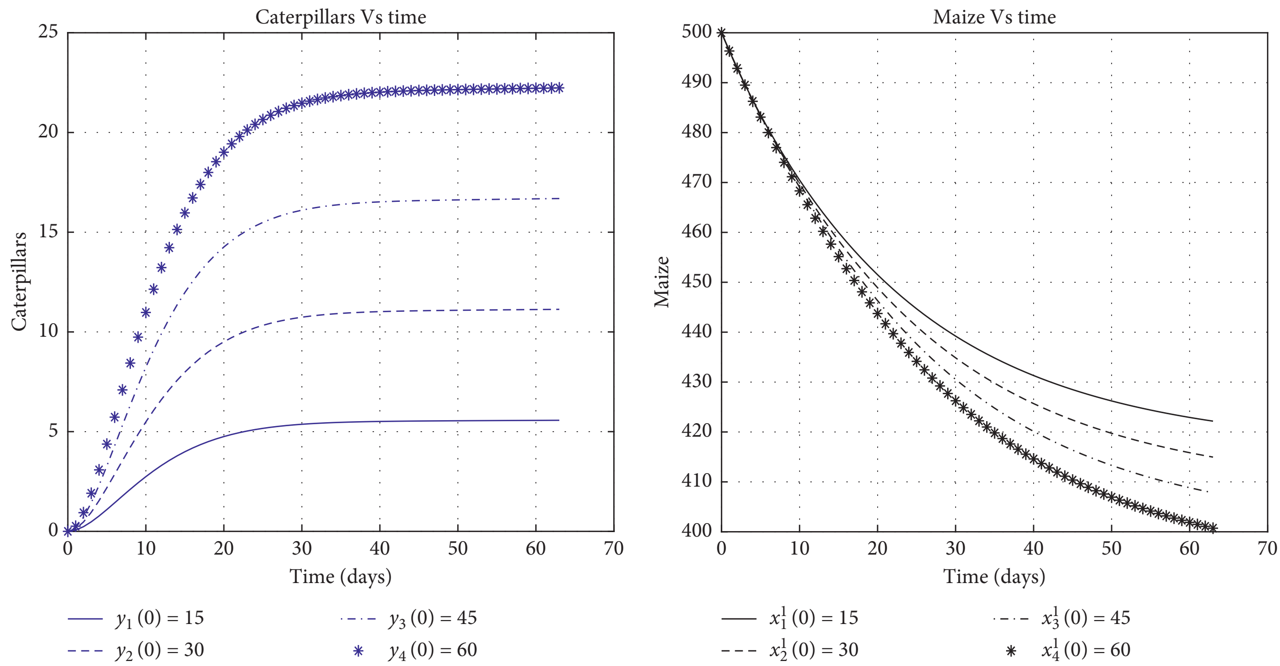


(a)



(b)

Figure 2: Continued.

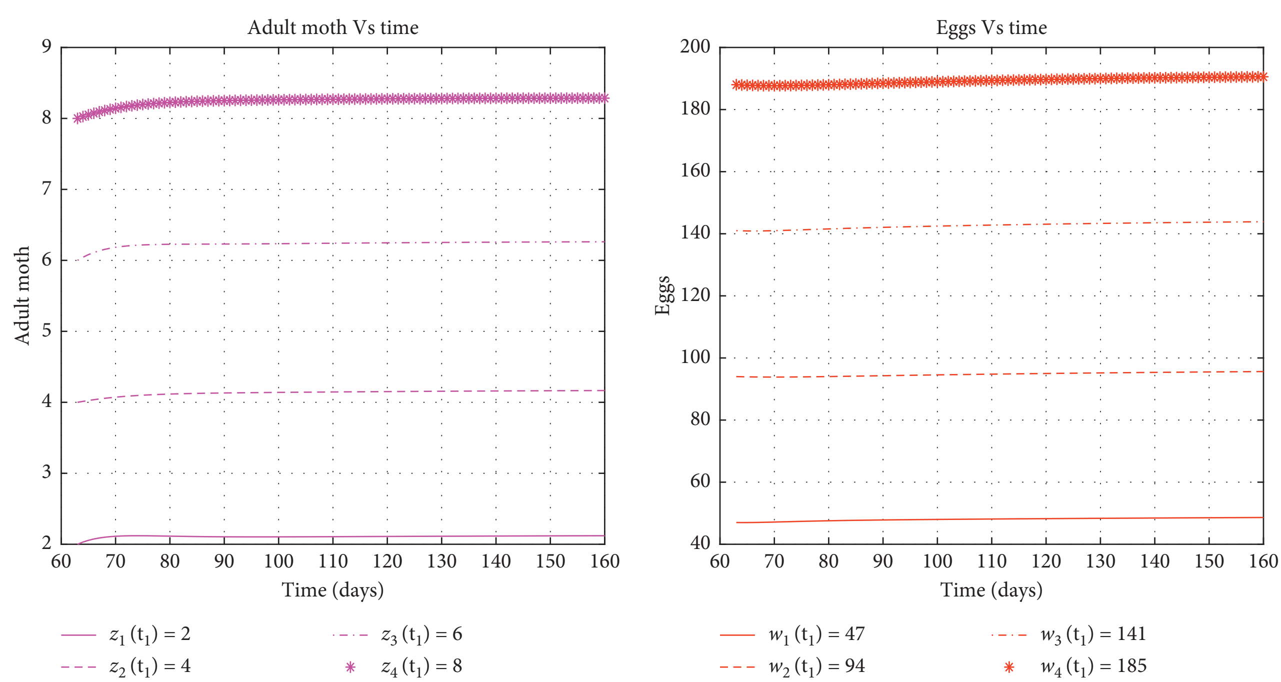


(c) (d)

Figure 2: Full simulation for the model system (1) with initial values in each of the subfigures (a–d) given in the legend.

Table 4: The numbers of moths, eggs, caterpillars, and maize densities when there is no immigration of the moth.

The initial number of the moths at $t = 0$	$z_1(0) = 15$	$z_2(0) = 30$	$z_3(0) = 45$	$z_4(0) = 45$
Adult moths	2	4	6	8
Eggs	47	94	141	185
Caterpillars	6	11	17	22
Maize plants	422	415	408	401



(a) (b)

Figure 3: Continued.

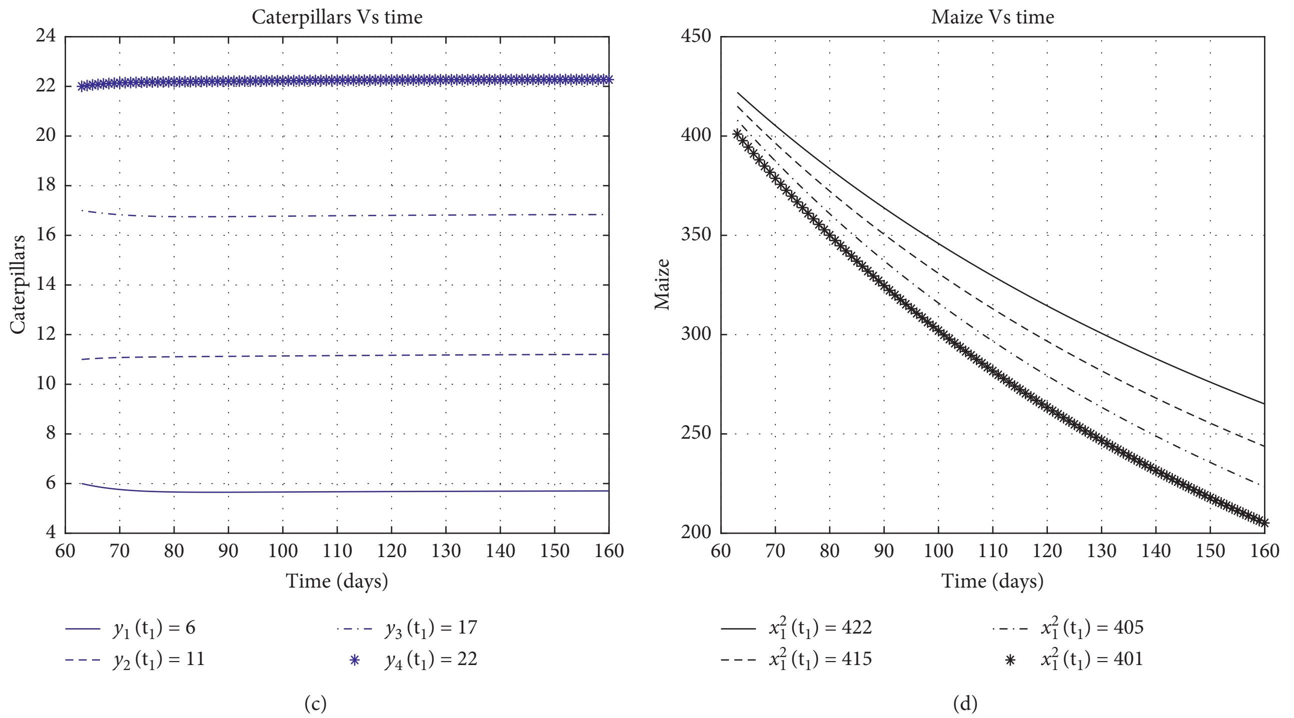


Figure 3: Full simulation for the model system (3) with initial values in each of the subfigures (a–d) given in the legend.

Table 5: Numbers of moths, eggs, caterpillars, and maize densities when there is no immigration of the moth.

Maize		Adult moths		Caterpillars		Eggs	
$x_2(t_1)$	$x_2(t_2)$	$z(t_1)$	$z(t_2)$	$y(t_1)$	$y(t_2)$	$w(t_1)$	$w(t_2)$
422	265	2	2	6	6	47	8
415	243	4	4	11	11	94	185
408	223	6	6	17	17	141	22
401	205	8	8	22	22	185	401

In the table, $x_2(t_1) = x_1(t_1)$, $z(t_2) = z(t_1)$, $y(t_2) = y(t_1)$, and $w(t_2) = w(t_1)$ for $t_1 \leq t \leq t_2$.

damaging stage of the moth. The simulation results for the moth visitation are shown in Figures 4(a)–4(d).

From Figure 4, the results indicate that immigration of the adult moths into maize field at different rates (i.e., $\sigma_1 = 15$, $\sigma_2 = 30$, $\sigma_3 = 45$, and $\sigma_4 = 60$) reproduces prolifically and reaches its environmental carrying capacity which also increases eggs production and density of the caterpillars to its environmental carrying within 63 days (see Figures 4(a)–4(c)). This increase adversely affects maize growth and exponentially decreases the biomass of these plants. Figures 4(a)–4(d) also justify that there is a delay of the caterpillar in damaging maize from $t = 0$ to $t = 10$ days. During this period, the adult moth lays eggs and the eggs hatch into caterpillars. On the other hand, maize plants grow keeping up their carrying capacity ($k = 500$), where after $t = 10$ days the population size for maize declines abruptly for each immigration rate. Table 6 shows the different immigration rates and their corresponding increase in eggs production, density of caterpillar, and the impact on maize growth in 63 days.

However, when the results for the model system (16) displayed in Table 6 are used as initial values in simulating model system (18) from $t_1 = 63$ to $t_2 = 160$ days, we obtain the results shown in Figure 5.

The results show that the numbers of adult moths, egg production, and density of caterpillars are at equilibrium (Figures 5(a)–5(c)) but continue decreasing for each number of the adult moths assumed to immigrate due to the existence of the caterpillars in the population (Figure 5(d)). The numbers of adult moths, egg production, and density of caterpillars at equilibrium as well as the corresponding effects on maize growth for each immigration rate assumed in 97 days are displayed in Table 7.

We generally conclude by comparing the results from Figures 2–5 based on two criteria: population dynamics of the pest and the number of maize plants remaining in both vegetative and reproductive stages. From Figure 2(a), we observe that the number of moths declines very abruptly due to egg-laying process and mortality rate, while the numbers of eggs laid and caterpillars as we see in Table 4 and Figures 2(b) and 2(c) are progressively increasing up to time $t = 20$ days before attaining their equilibrium values, which continues even after $t = 63$ days as we see in Figures 3(a)–3(c). Referring to Tables 4 and 5, the increase in egg production and caterpillar density exponentially decrease the maize plant biomass, an effect which gives an average of 412 and 234 maize plants in vegetative and reproductive stages, respectively.

However, Figures 4(a)–4(c) and Table 6 indicate that the density of caterpillars, moths, and eggs production are progressively increasing to days before attaining their equilibrium values which appear to continue even after $t =$

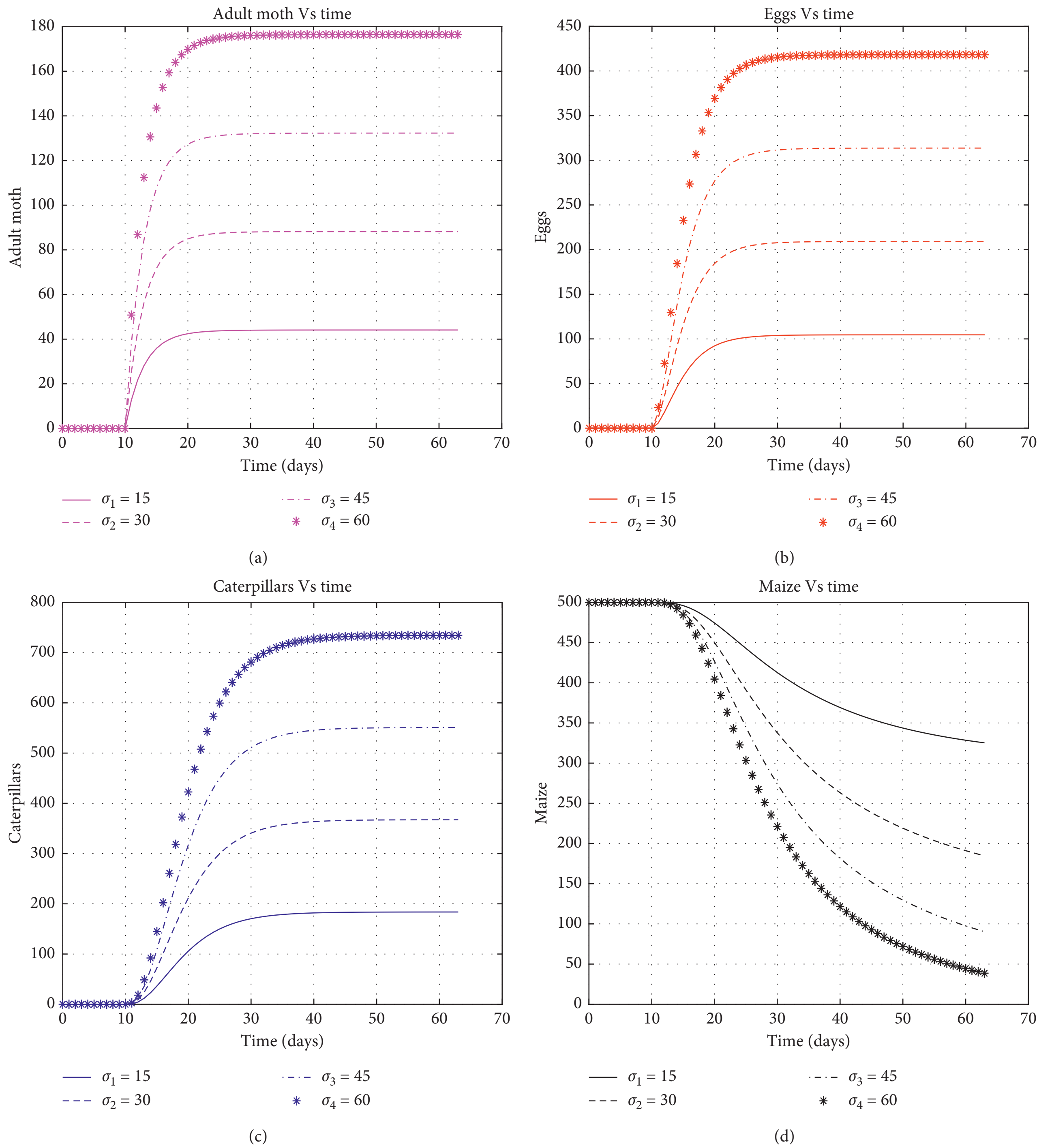


FIGURE 4: Simulation of the model system (16) demonstrated for different values given in Table 3 with initial values $x_1(0) = 500$, $y(0) = 0$, $z(0) = 0$, and $w(0) = 0$ from $t = 10$ to $t = 63$ and at different immigration rates: $\sigma_1 = 15$, $\sigma_2 = 30$, $\sigma_3 = 45$, and $\sigma_4 = 60$. (a) The adult moth immigrating into the maize field. (b) The increase in population size of the eggs laid after immigration. (c) The increase in the number of the caterpillars after the hatching process. (d) The decline in maize population size after immigration. Hence, results in each of the immigration rates at $t = 63$ days are the initial solutions for the model system (18).

63 days; see Figures 5(a)–5(c). This situation has a huge impact on maize biomass as we see in Figure 5(d) and Tables 6 and 7; the averages of maize plants remaining are 160 and 142 in the vegetative and reproductive stages, respectively, due to this effect. These comparisons give us a way to focus and decide where to introduce our control to reduce the effect, where, according to Section 4.5, immigration of the moths at $t > 0$ seems to be more destructive compared to the moth existing at $t = 0$ as seen in Section 4.4. In this regard, we introduce our controls to the model with the

TABLE 6: Numbers of moths, eggs, caterpillars, and maize densities due to immigration.

Immigration rates	$\sigma_1 = 15$	$\sigma_2 = 30$	$\sigma_3 = 45$	$\sigma_4 = 60$
Adult moths	44	88	132	176
Eggs	105	209	315	418
Caterpillars	184	367	550	734
Maize plants	329	185	90	39

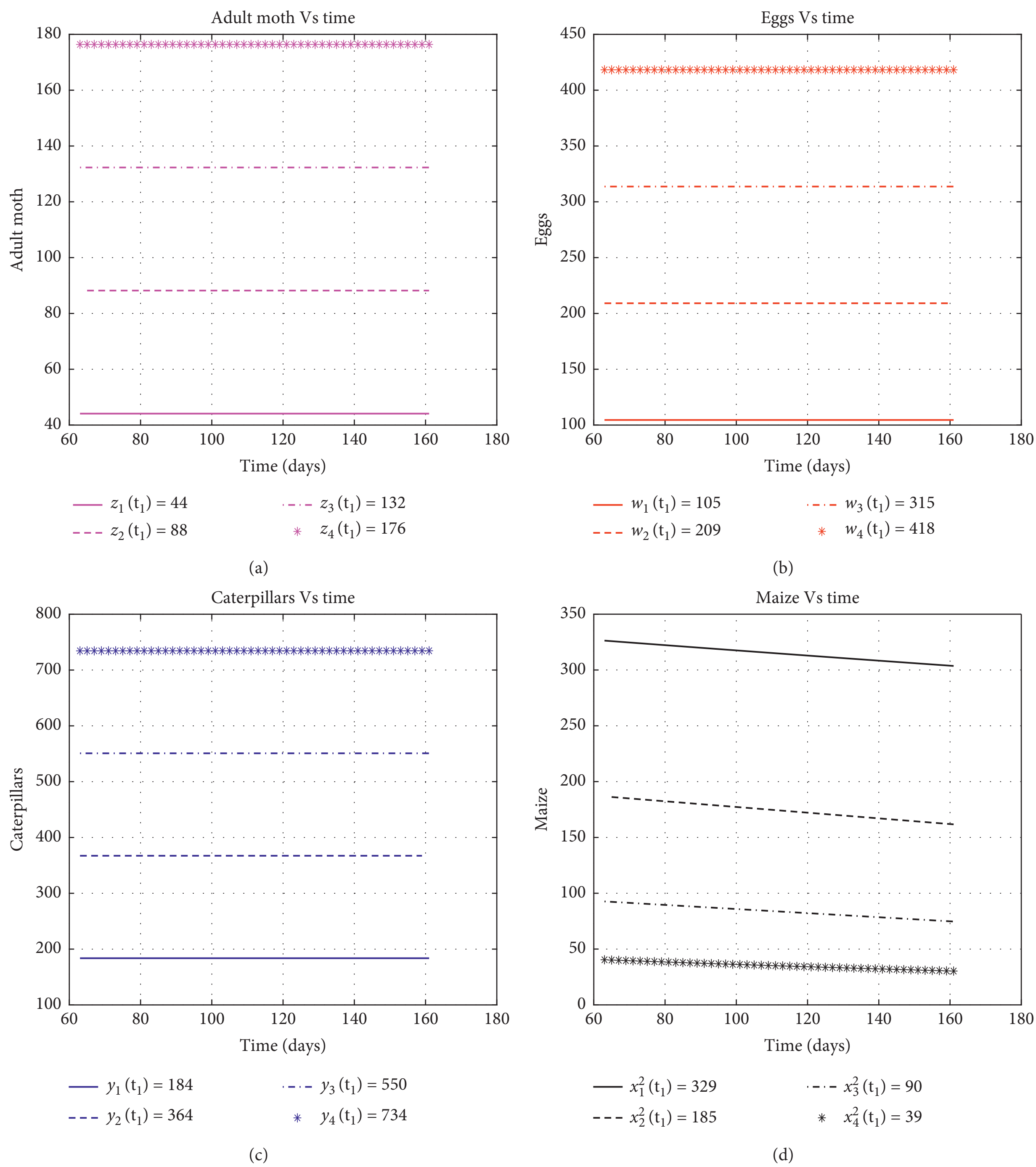


FIGURE 5: The numerical simulation for the model system (18) from $t = 63$ to $t = 160$ using the final results in Table 6 obtained in $t_1 = 63$ days. Figures 5(a)–5(c) maintain the equilibrium values for the number of moths, egg production, and density of caterpillars from the early stages of maize growth, while (d) indicates the reduction of maize for each immigration rate.

Table 7: Numbers of adult moths, eggs, caterpillars, and maize densities when there is no immigration of the moth.

Maize		Adult moths		Caterpillars		Eggs	
$x_2(t_1)$	$x_2(t_2)$	$z(t_1)$	$z(t_2)$	$y(t_1)$	$y(t_2)$	$w(t_1)$	$w(t_2)$
329	304	45	44	184	184	105	105.6
185	161.7	88.2	88	367	367	209	210
90	74.64	132.4	132	550	550	315	316
39	30.3	177	176	734	734	418	418.5

In the table, $x_2(t_1) = x_1(t_1)$, $z(t_2) = z(t_1)$, $y(t_2) = y(t_1)$, and $w(t_2) = w(t_1)$ for $t_1 \leq t \leq t_2$.

immigration of the moth to reduce the effects associated with this behavior.

5. Impact of Population Dynamics with Some Control Strategies

In the context of Integrated Pest Management (IPM), pest control strategies include a suitable combination of biological, cultural, and chemical control techniques, where most of these controls, for example, biological controls, were mathematically described by Paez [8] in terms of a smooth ordinary differential equation. Daudi [31] also identified the impossibilities of controlling the FAW using biological control. In this regard, our model shall bear a combination of controls, which includes harvesting effort (cultural methods) and spraying of pesticides. Harvesting is an important and effective cultural control method to prevent and control the explosive growth of the FAW in the maize field. Based on the Monitoring, Surveillance, and Scouting approach, harvesting is the removal of damaged cobs and stems from the field to reduce the number of caterpillars and thus infestation of the next crop [33].

There are two types of harvesting: nonzero constant and linear harvesting [34]. Due to the unexpected invasion of the number of moths which finally give us an abundant number of caterpillars, which always increases depending on the moth, only a linear harvesting rate will be incorporated in this model. Assuming that only the caterpillar which is unable to fly will be harvested in the field, we let h represent the catchability coefficient of the caterpillar at a given time. We further assume that the harvesting rate in both stages is proportional to the size of caterpillar population. Alongside this control, we introduce another control method of applying a pesticide to precede the linear harvesting.

According to Liang [13], a pesticide is a chemical or biological agent that kills the pest in a density-dependent manner. When introduced in the model, according to Liang [13], it affects the FAW due to chemical toxicity but has no direct effects on maize plants. We assume that pesticide control is more effective on both caterpillars and moths. We use model equations (16) and (18) as our baseline model systems to introduce both controls (pesticide and harvesting). Since we are interested in killing the caterpillar and the adult moth, we let u denote the strength of the chemical pesticide sprayed and let d_1 and d_2 denote the damage coefficients due to sprayed pesticide on the caterpillar and the moth, respectively. We also assume that spraying of the pesticide is done continuously and in both stages of maize growth. Below is the model with control parameters.

The model in the vegetative stage $(0 < t < t_1)$ is

$$\begin{aligned}
\frac{dx_1}{dt} &= -\alpha x_1 y - \lambda x_1, \\
\frac{dy}{dt} &= e_1 \alpha x_1 y + \gamma w - \delta y - \mu_y y - d_1 u y - h y, \\
\frac{dz}{dt} &= \delta y - \mu_z z + \sigma - d_2 u z, \\
\frac{dw}{dt} &= \rho z - \gamma w - \mu_w w,
\end{aligned} \tag{20}$$

subject to (2).

The model in the reproductive stage $(t_1 \leq t < t_2)$ is

$$\begin{aligned}
\frac{dx_2}{dt} &= -\eta x_2 y - \lambda x_2, \\
\frac{dy}{dt} &= e_2 \eta x_2 y + \gamma w - \delta y - \mu_y y - d_1 u y - h y, \\
\frac{dz}{dt} &= \delta y - \mu_z z + \sigma - d_2 u z, \\
\frac{dw}{dt} &= \rho z - \gamma w - \mu_w w,
\end{aligned} \tag{21}$$

subject to (4).

Since our problem is not based on a case study, for the simulation purposes, we make use of the data from the literature as seen in Table 3 as well as $d_1 = 3$, $d_2 = 11$, $u = 0.08$, and $h = 1.7861$, which were used by Paez and Hui [8, 35] to simulate the model systems (20) and (21) with controls being applied under the assumption that the moth is immigrating in the field at different rates per day such as $\sigma = 15, 30, 45, 60$ moths/day.

From the results shown in Figures 6–9, deploying pesticide and harvesting provides a satisfactory answer to our goal as the maximum control of the pest dynamics was achieved. Comparing the results in Tables 6 and 7, the caterpillar, eggs, and adult moths as we see in Figure 6(a) were unable to reach their environmental carrying capacity and were suppressed to 3, 16, and 17, respectively, with a resultant increase in maize plants to 492 within 63 days, while in Figure 6(b) the pest was suppressed to 3 caterpillars, 36 moths, and 31 eggs with a subsequent increase in maize plants to 446 in 97 days.

Furthermore, in Figure 7(a), the pest was reduced to 4 caterpillars, 36 moths, and 31 eggs and the maize plants increased to 491 in 63 days, while in Figure 7(b) the pest was decreased to 2 caterpillars, 37 moths, and 30 eggs and the maize plants peaked to 445 within 97 days. Moreover, Figure 8(a) shows that the caterpillars, adult moths, and eggs decreased to 5, 55, and 54, respectively, and the maize plants increased to 443, whereas in Figure 8(b) the pest was reduced to 4 caterpillars, 54 moths, and 47 eggs with a resultant increase in maize plants to 444. We finally observe in

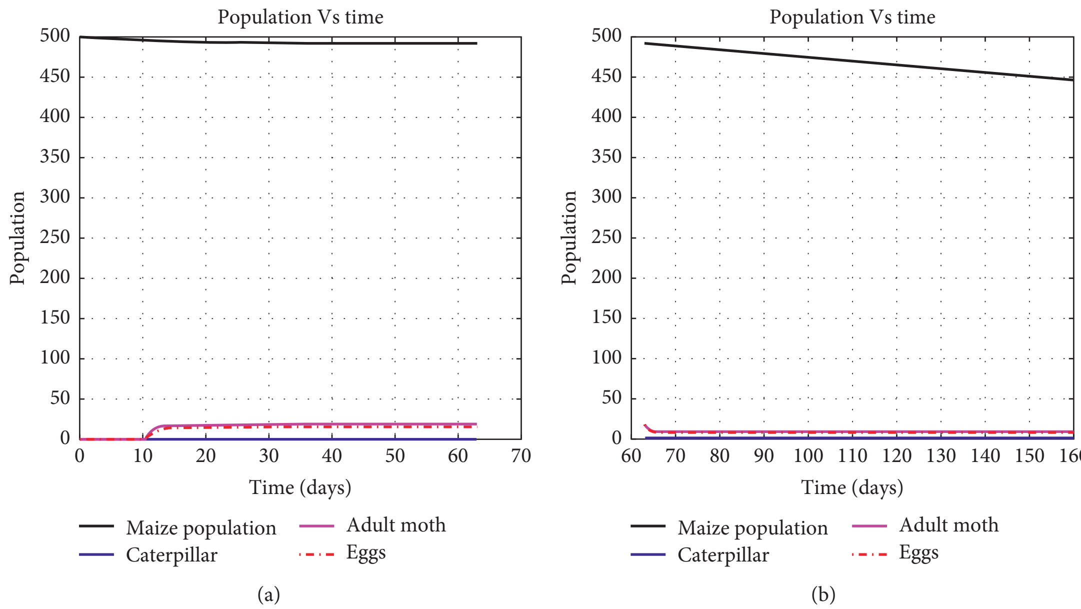


Figure 6: Effects of deploying pesticide and harvesting when 15 adult moths immigrate into the field after 10 days. Subfigure 6(a) shows the simulation of the model system (20) from time $t > 0$ to $t_1 = 63$ days with initial values $x_1(t) = 500$, $y(0) = 0$, $z(t) = 0$, and $w(t) = 0$. Subfigure 6(b) displays simulation of the model system (21) from time $t = 63$ to $t_2 = 160$ using results of subfigure 6(a) taken at $t_1 = 63$ as initial values.

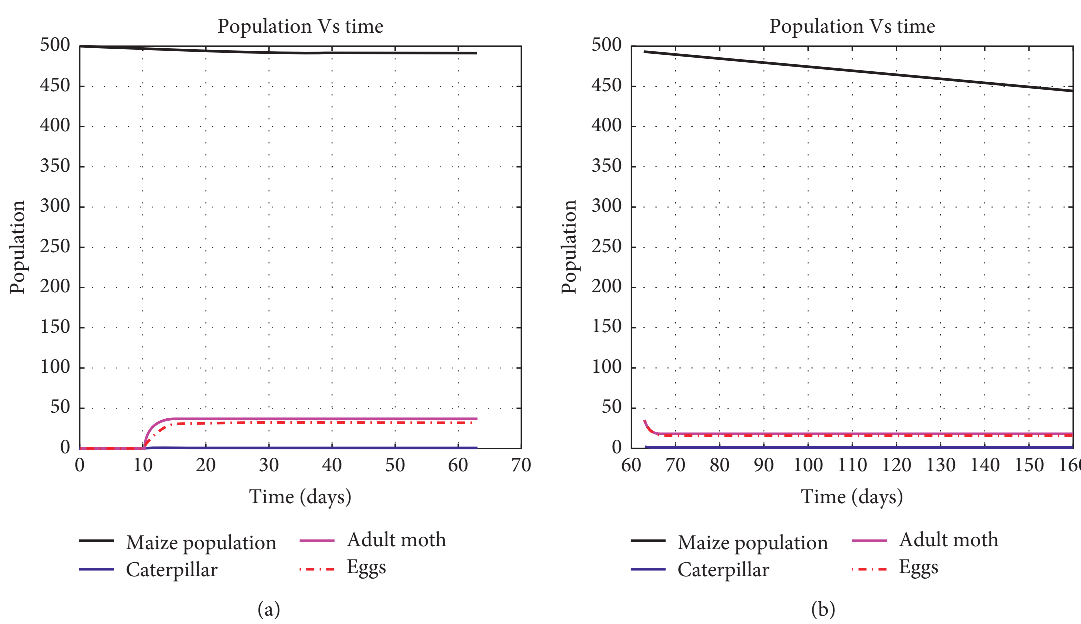
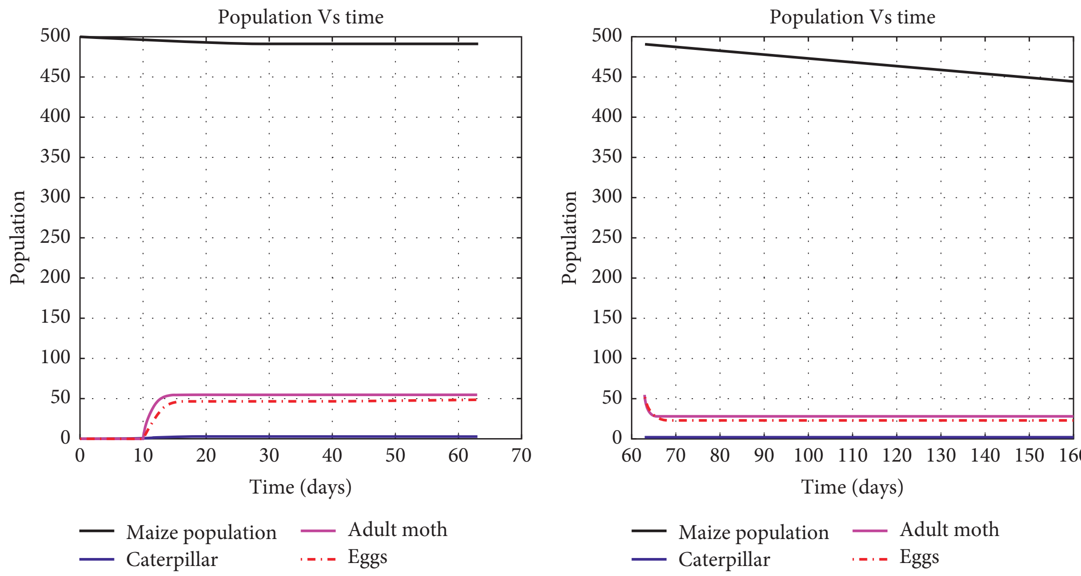


Figure 7: Effects of deploying pesticide and harvesting when 30 adult moths immigrate into the field after 10 days. Subfigure 7(a) shows the simulation of the model system (20) from time $t > 0$ to $t_1 = 63$ days with initial values $x_1(t) = 500$, $y(0) = 0$, $z(t) = 0$, and $w(t) = 0$. Subfigure 7(b) displays simulation of the model system (21) from time $t = 63$ to $t_2 = 160$ using results of subfigure 7(a) taken at $t_1 = 63$ as initial values.

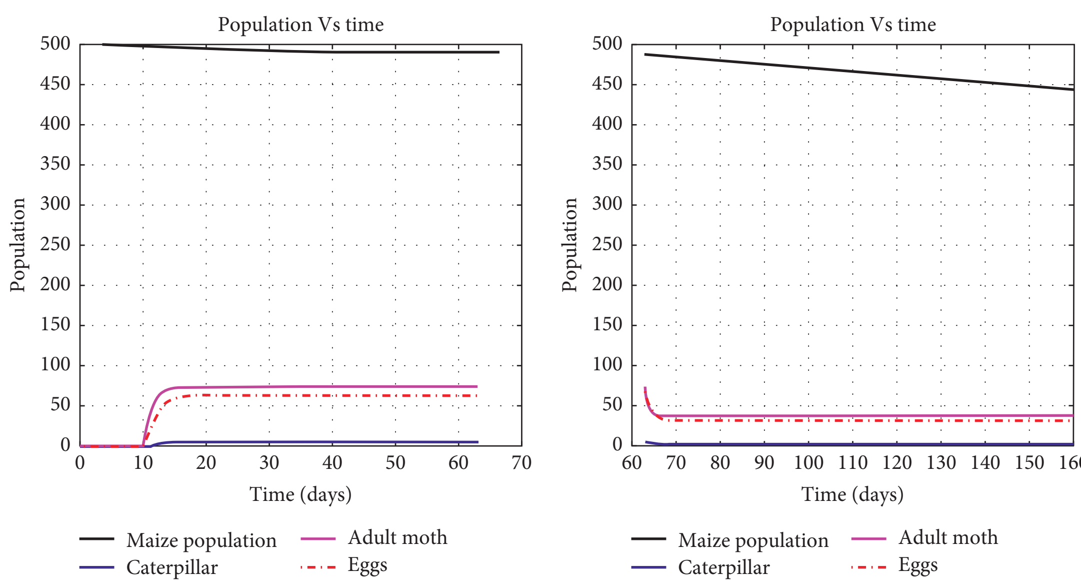
Figure 9(a) that the caterpillars, adult moths, and eggs were suppressed to 4, 72, and 62, respectively, and the maize plants increased to 449, while in Figure 9(b) the pest was reduced to 3 caterpillars, 54 moths, and 47 eggs with an increase in maize plants to 443.

The results in Figures 6–9 confirm that the suppression rate of the moths, eggs, and caterpillars in Figure 6 was more effective than that in Figures 7–9 due to the small number of the adult moths that immigrated into the field. It can also be seen from Figures 8 and 9 that the numbers of eggs and adult



(a) (b)

Figure 8: Effects of deploying pesticide and harvesting when 45 adult moths immigrate into the field after 10 days. Subfigure 8(a) shows the simulation of the model system (20) from time $t > 0$ to $t_1 = 63$ days with initial values $x_1(t) = 500$, $y(0) = 0$, $z(t) = 0$, and $w(t) = 0$. Subfigure 8(b) displays simulation of the model system (21) from time $t = 63$ to $t_2 = 160$ using results of subfigure 8(a) taken at $t_1 = 63$ as initial values.



(a) (b)

Figure 9: Effects of deploying pesticide and harvesting when 60 adult moths immigrate into the field after 10 days. Subfigure 9(a) shows the simulation of the model system (20) from time $t > 0$ to $t_1 = 63$ days with initial values $x_1(t) = 500$, $y(0) = 0$, $z(t) = 0$, and $w(t) = 0$. Subfigure 9(b) displays simulation of the model system (21) from time $t = 63$ to $t_2 = 160$ using results of subfigure 8(a) taken at $t_1 = 63$ as initial values.

moths remaining in the field after applying control were large compared to the numbers shown in Figures 6 and 7 due to a large number of moths immigrating ($\sigma = 45$ and $\sigma = 60$) into the field at time $t = 10$ days. We therefore conclude that it is essential to use surveillance and monitoring systems for timely control of the pest; and it is better to apply control once a small population of all pest life cycle stages have been established.

6. Discussion and Concluding Remark

In this paper, a generic model with two submodels for assessing the impact of the FAW infestation on the maize production in the vegetative and reproductive stages was formulated. The two submodels interconnected by the initial condition $x_1(t_1) = x_2(t_1)$ at the transitional period (i.e., $t = t_1$) were stage-structured in both populations of maize and FAW. The preliminary analysis of the two submodels revealed that the two systems had a unique and positively bounded solution for all time $t \geq 0$. A numerical analysis of the submodels was also carried out under two different cases. Case 1 concerns different number of adult moths in the field which were assumed at $t = 0$ (with no immigration) and Case 2 concerns the existence of exogenous factors that lead to the immigration of the adult moths into the field at a time t.

The results indicated that the destruction of maize biomass which is accompanied by the decrease in maize plants to an average of 160 and 142 in the vegetative and reproductive stages, respectively, was observed to be higher in Case 2 than in Case 1 due to the subsequent increase in egg production and density of the caterpillars in a few (10) days after immigration. This severe effect of maize biomass caused by the unprecedented numbers of caterpillars, eggs, and adult moths in the field after adult moths' immigration gave us an idea of extending the two submodels to include controls such as pesticide and harvesting. The results indicated that the proposed approach significantly suppressed the numbers of caterpillars, eggs, and adult moths with a resultant increase in maize plants to averages of 467 and 443 in the vegetative and reproductive stages, respectively.

This work is not exhaustive as we expect in the future to deploy optimal control of the FAW on maize plants to minimize not only the cost of using pesticides and harvesting but also the side effects on human health and the environmental impact. This approach will also help to determine the maximum sustainable yield of maize crops and improve the quantity and quality of food to farmers.

Data Availability

All the data used are obtained from different studies, which have been cited in the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

S. Daudi contributed to formulation of two stage-structured submodels and analysis. L. Luboobi, M. Kgosimore, and D. Kuznetsov contributed to supervision, writing, and review of the paper.

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References

[1] United Nations, "Concise report on the world population situation in 2014," 2014.

[2] B. Shiferaw, M. Smale, H.-J. Braun, E. Duveiller, M. Reynolds, and G. Muricho, "Crops that feed the world 10. Past successes and future challenges to the role played by wheat in global food security," *Food Security*, vol. 5, no. 3, pp. 291–317, 2013.

[3] S. Bhusal and E. Chapagain, "Threats of fall armyworm (Spodoptera frugiperda) incidence in Nepal and it's integrated management-A review," *Journal of Agriculture and Natural Resources*, vol. 3, no. 1, pp. 345–359, 2020.

[4] H. De Groote, S. C. Kimenju, B. Munyua, S. Palmas, M. Kassie, and A. Bruce, "Spread and impact of fall armyworm (Spodoptera frugiperda J.E. Smith) in maize production areas of Kenya," *Agriculture, Ecosystems & Environment*, vol. 292, 2020.

[5] F. Faithpraise, J. Idung, C. Chatwin, R. Young, and P. Birch, "Modelling the control of African Armyworm (Spodoptera exempta) infestations in cereal crops by deploying naturally beneficial insects," *Biosystems Engineering*, vol. 129, pp. 268–276, 2015.

[6] R. Anguelov, C. Dufourd, and Y. Dumont, "Mathematical model for pest-insect control using mating disruption and trapping," *Applied Mathematical Modelling*, vol. 52, pp. 437–457, 2017.

[7] M. Kebede and T. Shimalis, "Out-break, distribution and management of fall armyworm, Spodoptera frugiperda JE Smith in Africa: the status and prospects," *Applied Mathematical Modelling*, vol. 3, pp. 551–568, 2018.

[8] J. Páez Chávez, D. Jungmann, and S. Siegmund, "Modeling and analysis of integrated pest control strategies via impulsive differential equations," *International Journal of Differential Equations*, vol. 2017, no. 18, pp. 1–18, 2017.

[9] I. G. Pearce, M. A. J. Chaplain, P. G. Schofield, A. R. A. Anderson, and S. F. Hubbard, "Modelling the spatio-temporal dynamics of multi-species host-parasitoid interactions: heterogeneous patterns and ecological implications," *Journal of Theoretical Biology*, vol. 241, no. 4, pp. 876–886, 2006.

[10] R. Early, P. González-Moreno, S. T. Murphy, and R. Day, "Forecasting the global extent of invasion of the cereal pest Spodoptera frugiperda, the fall armyworm," *NeoBiota*, vol. 40, no. 25, pp. 25–50, 2018.

[11] R. Day, P. Abrahams, M. Bateman et al., "Fall armyworm: impacts and implications for Africa," *Outlooks on Pest Management*, vol. 28, no. 5, pp. 196–201, 2017.

[12] S. Bista, M. K. Thapa, and S. Khanal, "Fall armyworm: menace to Nepalese farming and the integrated management approaches," *International Journal of Environment, Agriculture and Biotechnology*, vol. 5, no. 4, pp. 1011–1018, 2020.

[13] J. Liang, S. Tang, and R. A. Cheke, "An integrated pest management model with delayed responses to pesticide applications and its threshold dynamics," *Nonlinear Analysis: Real World Applications*, vol. 13, no. 5, pp. 2352–2374, 2012.

[14] G. Murúa, J. Molina-Ochoa, and C. Coviella, "Population dynamics of the fall armyworm, Spodoptera frugiperda (Lepidoptera: noctuidae) and its parasitoids in northwestern Argentina," *Florida Entomologist*, vol. 89, no. 2, pp. 175–182, 2006.

[15] M. Rafikov, J. M. Balthazar, and H. F. Von Bremen, "Mathematical modeling and control of population systems: applications in biological pest control," *Applied Mathematics and Computation*, vol. 200, no. 2, pp. 557–573, 2008.

[16] X.-a. Zhang, L. Chen, and A. U. Neumann, "The stage-structured predator-prey model and optimal harvesting policy," *Mathematical Biosciences*, vol. 168, no. 2, pp. 201–210, 2000.

[17] M. Bandyopadhyay and S. Banerjee, "A stage-structured prey-predator model with discrete time delay," *Applied Mathematics and Computation*, vol. 182, no. 2, pp. 1385–1398, 2006.

[18] B. Kang, M. He, and B. Liu, "Optimal control of agricultural insects with a stage-structured model," *Mathematical Problems in Engineering*, vol. 2013, no. 8, pp. 1–8, 2013.

[19] F. Pfab, M. V. R. Stacconi, G. Anfora, A. Grassi, V. Walton, and A. Pugliese, "Optimized timing of parasitoid release: a mathematical model for biological control of Drosophila suzukii," *Theoretical Ecology*, vol. 11, no. 4, pp. 489–501, 2018.

[20] W. G. Aiello, H. I. Freedman, and J. Wu, "Analysis of a model representing stage-structured population growth with state-dependent time delay," *SIAM Journal on Applied Mathematics*, vol. 52, no. 3, pp. 855–869, 1992.

[21] S. Kumar, S. Ahmad, M. I. Siddiqi, and K. Raza, "Mathematical model for Plant-Insect interaction with dynamic response to PAD4-BIK1 interaction and effect of BIK1 inhibition," *Biosystems*, vol. 175, pp. 11–23, 2019.

[22] W. G. Aiello and H. I. Freedman, "A time-delay model of single-species growth with stage structure," *Mathematical Biosciences*, vol. 101, no. 2, pp. 139–153, 1990.

[23] R. Shi and L. Chen, "Staged-structured Lotka-Volterra predator-prey models for pest management," *Applied Mathematics and Computation*, vol. 203, no. 1, pp. 258–265, 2008.

[24] T. K. Kar and H. Matsuda, "Global dynamics and controllability of a harvested prey-predator system with Holling type III functional response," *Nonlinear Analysis: Hybrid Systems*, vol. 1, no. 1, pp. 59–67, 2007.

[25] F. Assefa and D. Ayalew, "Status and control measures of fall armyworm (Spodoptera frugiperda) infestations in maize fields in Ethiopia: a review," *Cogent Food & Agriculture*, vol. 5, no. 1, 2019.

[26] J. W. Chapman, T. Williams, A. M. Martínez et al., "Does cannibalism in Spodoptera frugiperda (Lepidoptera: noctuidae) reduce the risk of predation?" *Behavioral Ecology and Sociobiology*, vol. 48, no. 4, pp. 321–327, 2000.

[27] W. Aloyce, D. Kuznetsov, and L. S. Luboobi, "A mathematical model for the MLND dynamics and sensitivity analysis in a maize population," *Asian Journal Of Mathematics And Applications*, vol. 2017, no. 9, pp. 1–19, 2017.

[28] L. T. Li, Y. Q. Wang, J. F. Ma et al., "The effects of temperature on the development of the moth Athetis lepigone, and a prediction of field occurrence," *Journal of Insect Science*, vol. 13, no. 3, pp. 103–106, 2013.

[29] J. Tumwiine, L. S. Luboobi, and J. Y. T. Mugisha, "Modeling the effect of treatment and mosquito control on malaria transmission," *International Journal of Management and Systems*, vol. 21, no. 2, pp. 107–124, 2005.

[30] H. T. Alemneh, O. D. Makinde, and D. Mwangi Theuri, "Ecoepidemiological model and analysis of MSV disease transmission dynamics in maize plant," *International Journal of Mathematics and Mathematical Sciences*, vol. 2019, no. 4, pp. 1–14, 2019.

[31] S. Daudi, L. Luboobi, M. Kgosimore, D. Kuznetsov, and S. Mushayabasa, "A mathematical model for fall armyworm management on maize biomass," *Advances in Difference Equations*, vol. 3, 2021.

[32] K. M. Putri, "June. A maize foliar disease mathematical model with standard incidence rate," 2019.

[33] I. U. Khan, M. Nawaz, S. Fazal, and S. Kamran, "Integrated pest management of maize stem borer, Chilo partellus (Swinhoe) in maize crop and its impact on yield," *Journal of Entomology and Zoology Studies*, vol. 3, no. 5, pp. 470–472, 2015.

[34] T. K. Kar and U. K. Pahari, "Modelling and analysis of a prey-predator system with stage-structure and harvesting," *Nonlinear Analysis: Real World Applications*, vol. 8, no. 2, pp. 601–609, 2007.

[35] J. Hui and D. Zhu, "Dynamic complexities for prey-dependent consumption integrated pest management models with impulsive effects," *Chaos, Solitons & Fractals*, vol. 29, no. 1, pp. 233–251, 2006.